EFFECTS OF MULTIPLE SCATTERING FOR MILLIMETER-WAVELENGTH WEATHER RADARS

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ABSTRACT

Effects of multiple scattering on the reflectivity measurement for millimeter-wavelength weather radars are studied, in which backscattering enhancement may play an important role. In the previous works, the backscattering enhancement has been studied for plane wave injection, the reflection of which is received at the infinite distance. In this paper, a finite beam width of a Gaussian antenna pattern along with spherical wave is taken into account. A time-independent second order theory is derived for a single layer of clouds of a uniform density. The ordinary second-order scattering (ladder term) and the second-order backscattering enhancement (cross term) are derived for both the copolarized and cross-polarized waves. As the optical thickness of the hydrometeor layer increases, the differences from the conventional plane wave theory become more significant, and essentially the reflectivity of multiple scattering depends on the ratio of mean free path in hydrometeors to radar footprint radius. This effect must be taken into account for remote sensing applications.

1. INTRODUCTION

From the early 1970's to the early 1990's, multiple scattering in randomly distributed particles was intensively studied through the analytical method of electromagnetic wave (de Wolf, 1971; Golubentsev, 1984; Kravtsov and Saichev, 1982; Kuga and Ishimaru, 1984; Tsang and Ishimaru, 1985; Tsang and Kong, 2001). In the course of study, two main contributions of multiple scattering to reflective intensity were revealed. In general, the electromagnetic field reflected from a random medium can be represented as a sum of fields from many portions of the medium. Among this sum, only pairs of fields that have strong correlation can give contributions to a measured intensity. A first possible pair is constituted of a field E_A and its self-complex-conjugate field E_A^* as depicted in Fig. 1a in the case of the second order scattering. The incident field E_A emits from the antenna T, and scatters at points 'b' and 'a' successively in the random medium, returning to the antenna T. This process is represented by the ladder diagram in Fig. 1b, which can be proven to be the basis of radiative transfer theory (Tsang and Kong, 2001). Another possible pair occurs in the backscattering condition depicted in Fig. 2a, in which the field E_A travels in the same path as that in Fig. 1a, while the conjugate field E_B, takes the reversal path of E_A. This process can be represented by the cross diagram in Fig. 2b. Since the path lengths of E_A and E_B are equal for the right backscattering, it always gives finite contribution to a measured intensity. As the scattering angle deviates from the right backscattering condition, the fields E_A and E_B have different path lengths, and random distributions of particles ‘a’ and ‘b’ will cause strong decorrelation, generally giving negligible contribution to the measured intensity. In short, the cross term is measured only in the vicinity of the right backscattering angle, which is the reason we refer to this phenomenon as backscattering enhancement.

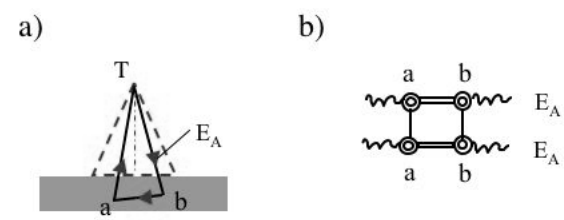

Figure 1. Second order ladder term. An incident field E_A emits from the antenna T, and scatters at points 'b' and 'a' successively in the random media, returning to the antenna T. (a): Geometry. (b): Diagram.

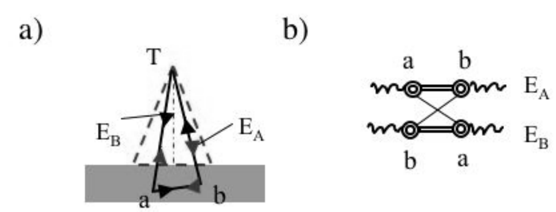

Figure 2. Second order cross term. An incident field E_A takes the same path as in Fig. 1a, while the conjugate field E_B emits from the antenna T, and scatter at points 'a' and at 'b' successively (i.e. time-reversal path of E_A), returning to the antenna T. (a): Geometry. (b): Diagram.

In all the previous theoretical works, a plane wave is injected to randomly distributed particles, and the reflected wave is collected by a receiver at infinite range. On the other hand, in remote sensing, a spherical wave with a finite beam width, usually approximated as a Gaussian antenna pattern, is injected, and the reflected wave is received by an antenna at a finite range. For the single scattering, the plane wave theory can be applied to a radar of finite beam width along with slight corrections concerning to range and gain. While for multiple scattering, we can not adopt the plane wave theory, because a finite size footprint can be considered to give a smaller reflectivity than the plane wave theory predicts, especially when the footprint size is much smaller than the mean free path of an illuminated body. The mean free path in a layer of hydrometeors often reaches of the order of 1000 meters for millimeter wavelength wave, while typical foot print sizes are of order of hundreds meters. In this study, a time-independent multiple scattering theory is formulated for a spherical wave along with a Gaussian antenna pattern, based on the plane wave theory of Mandt et al.(1990). Our analysis considers only a single layer of spherical water particles of a uniform diameter and a uniform number density.

2. Formalism

In this section, the formalism of Ishimaru and Tsang (1988) and Mandt et al. (1990) is expanded to a spherical wave with a finite beam width represented by a Gaussian function. To deal with complication introduced by the finite beam width, further simplifications of integrals are performed for the second order ladder and cross terms.

We shall consider a single layer of hydrometeors of thickness *d* constituted of spherical water particles of a uniform diameter *D* and a uniform density N_0. A 3-dB beam width θ_d and a range r_s are assumed to be $|\theta_d| << 1$ and $r_s >> d$, respectively. An incident direction $\hat{k}_i$ is almost parallel to the scattering direction $\hat{k}_s$ ($\hat{k}_i \approx -\hat{k}_s$). The wavenumber in the medium is calculated self-consistently through the Fold-Twersky approximation. The 3-dB footprint radius σ_r can be defined as

$$\sigma_r^2 = r_s^2 \theta_d^2 / 2^3 \ln 2 \tag{1.1}$$

The complete forms and derivations up to the second order terms are skipped due to the limit of allowed pages, and interested readers should refer to Kobayashi et al. (2005). Instead, some characteristic features in the formulations will be mentioned. The derived cross term has a real valued form rather than a complex valued form described in Mandt et al. (1990). It also includes an oscillation term proportional to

$$\exp[i(k_{dz} + t)\zeta] \tag{1.2}$$

in which ς represents the relative coordinate in the vertical direction from a random point to another point in the medium, and the deviation vector $\mathbf{k_d}$ and the variable t have been introduced based on definitions by Tsang and Ishimaru (1985):

$$\mathbf{k_d} \equiv k_{dx}\hat{x} + k_{dy}\hat{y} + k_{dz}\hat{z} = k(\hat{k}_i + \hat{k}_s) \tag{1.3}$$

$$t \equiv k_{dx} \tan\theta \cos\varphi + k_{dy} \tan\theta \sin\varphi \tag{1.4}$$

In Eq. (1.4), θ and φ represent relative spherical coordinates between the two random points. Equation (1.2) means that decorrelation caused by random particles becomes more serious as deviation from the right backscattering angle increases. The condition of finite beam width brings the following exponential term into both the ladder and cross terms:

$$\exp[-\varsigma^2 \tan^2\theta / 4\sigma_r^2] \tag{1.5}$$

Equation (1.5) will play an important role when the mean free path is large comparing to a radar footprint size.

3. Results

When hydrometeors consist of spherical particles, the first order ladder term $I_L^{(1)}$ has only the copolarized component, i.e. $I_L^{(1)} = I_L^{(1)}(CO)$. Furthermore $I_L^{(1)}$ is almost constant in the vicinity of $\theta_s = 0$ ($\theta_s < 0.3$ degree), within which the backscattering enhancement occurs. For this reason, the intensity of a multiple scattering term will be normalized by $I_L^{(1)}$ to be converted to an effective reflectivity in the rest of paper.

The sums of only the second order terms $L_2^{co} + C_2^{co}$ in copolarization and $L_2^{cx} + C_2^{cx}$ in cross-polarization are plotted with the solid and dashed lines respectively in Fig. 3 as functions of the foot print radius normalized by the mean free path $l_0 = 1/\kappa_e$. Since the monostatic radar is our main concern, only the backscattering $\theta_s = 0$ will be considered hereinafter. Spherical water particles of a uniform diameter $D = 1$ mm with a particle number density $N_0 = 5 \times 10^3 \; m^{-3}$ are used for calculation along with a frequency of 95 GHz, which gives the mean free path $l_0 = 77.2$ m. Figure 3 shows that the reflectivities rapidly decrease in the region $\sigma_r / l_0 < 1$, while these are almost constant in the region $\sigma_r / l_0 > 2$.

In Fig. 4, the terms $L_2^{co} + C_2^{co}$ (solid lines) and $L_2^{cx} + C_2^{cx}$ (dashed lines) are plotted as a function of optical thickness $\tau_d = d / l_0$ for several values of normalized footprint radius σ_r / l_0. Here, only the frequency of 95 GHz and the particle diameter $D = 1$ mm are fixed, while the particle number density N_0 can be chosen arbitrarily. Since the mean free path l_0 is uniquely defined for a certain N_0 with the fixed *D*, the value of τ_d should be considered to be changed by varying the physical layer thickness *d*. Alternatively if a layer thickness *d*, and a particle number density N_0 are known with information of the particle diameter $D = 1$ mm, we can determine the reduction factor from the plane wave theory (i.e. $\sigma_r / l_0 = \infty$) to a finite footprint radius σ_r, using Fig. 4. Figure 4 also indicates the following fact: As the optical thickness of the hydrometeor layer increases, the differences from the conventional plane wave theory become more significant, and essentially the reflectivity of multiple scattering depends on the ratio of mean free path in hydrometeors to radar footprint radius. Inversely saying, the plane wave theory can be applied to a smaller value of σ_r / l_0, as the optical thickness decreases. For instance, the reflectivity for $\sigma_r / l_0 = 1$ can be approximated with the plane wave theory ($\sigma_r / l_0 = \infty$) at $\tau_d = 1$.

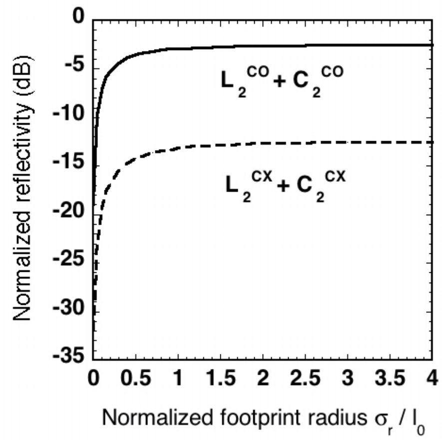

Figure 3. The reflectivity $L_2^{co} + C_2^{co}$ in copolarization (solid line) and the reflectivity $L_2^{cx} + C_2^{cx}$ in cross-polarization (dashed line) as functions of the normalized footprint radius σ_r / l_0 for the backscattering $\theta_s = 0$. Spherical water particle of diameter $D = 1$ mm and particle number density $N_0 = 5 \times 10^3 \ m^{-3}$ are used, which give the mean free path $l_0 = 77.2$ m.

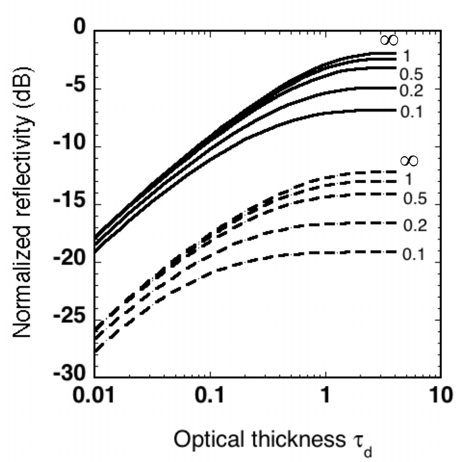

Figure 4. The reflectivity $L_2^{co} + C_2^{co}$ in copolarization (solid lines) and the reflectivity $L_2^{cx} + C_2^{cx}$ in cross-polarization (dashed lines) as functions of the optical thickness τ_d for the backscattering $\theta_s = 0$. Hydrometeor diameter is set at $D = 1$ mm. Particle number density N_0 is arbitrary. The parameters of footprint radius σ_r / l_0 are set at ∞, 1, 0.5, 0.2 and 0.1 from top to down. The differences of finite footprint radius (σ_r / l_0 =1-0.1) from the plane wave theory ($\sigma_r / l_0 = \infty$) reduce, as τ_d decreases.

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